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# Selection of the best-fit probability distribution for Brisbane River Catchment

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**Abstract:** *Flood is one of the worst natural hazards worldwide. Brisbane, Queensland, has experienced many severe flood events causing damages worth billions of dollar and human deaths. Accurate estimation of design floods with less uncertainty helps to minimise flood risk, damage and loss of human life. Among many design flood estimation methods, flood frequency analysis (FFA) is a widely used method. The primary step in FFA is the selection of a suitable probability distribution that fits the observed flood data adequately. As it is still not possible to select the best fit distribution from a large number of candidate distributions and associated parameter estimation procedures for a particular catchment with certainity, selection of probability distribution is still remain a difficult task. This study examines the selection of the best fit probability distribution for FFA. Brisbane River catchment of Queensland is selected as the study area. The annual maximum (AM) flood data from 26 stream gauging stations are selected with AM flood record lengths ranging from 20 to 91 years with a mean value of 47 years. Five different probability distributions and three goodness-of-fit tests are adopted. Based on a relative scoring method, Log Pearson Type III is found to be the most suitable probability distribution, followed by Generalised Pareto for the study area. To investigate the impact of high floods on the selection of the best fit probability distribution and flood quantile estimation, FFA are carried out twice; with the high flood values being included in the data and excluded from the data. It is found that the best fit probability distribution changes and magnitude of flood quantiles reduces notably if high floods are excluded from the data series.*

***Keywords**: Flood; Goodness-of-fit tests; Flood frequency analysis; Probability distribution.*

## 1. Introduction

Floods impact on both individuals and communities, and have notable social, economic, and environmental consequences (OQCS, 2016). Brisbane, the state capital of Queensland, Australia, experienced many dangerous floods including 2011 flood. Flood risk can be minimised through more accurate estimaiton of flood magnitude and frequecny of occurance of the flood. There are various methods available for flood estimation. Most of these methods of flood estimation use some types of probability concept including fitting some well established probability distribution with sample data. Flood frequency analysis is a statistical technique which fits a probability distribution to recorded streamflow data observed at a given location within a catchment (Haddad and Rahman, 2008). The fitted probability distribution is used for predictions of events beyond the range of the observed data period. If adequate and quality data is available, Australian Rainfall and Runoff (ARR) (Ball et al., 2016) recommend using at-site flood frequency analysis (FFA) for estimation of design peak floods.

Many different probability distributions and parameter estimation methods have been tested and recommended around the globe (Cunnane, 1989). Many probability distributions are available for modeling annual maximum (AM) flood series. Some of the commonly used distributions are Log Pearson Type 3 (LP3), General Extreme Value (GEV), Generalised Pareto (GP) Normal, Log Normal (LN), Pearson Type 3 (P3), Gamma, Extreme Value Type 1 (EV1), Extreme Value Type 2 (EV2), Two component Extreme Value, Exponential, Weibull and Wakeby (Cunnane 1989; Bobee et al. 1993). Many studies have been carried out by different researchers to find the appropriate probability distribution model for design flood estimation using FFA method (e.g., Alam et al., 2014; Rahman A S. et al. 2013; Haddad abd Rahman., 2008; Laio et al.,2009; Stedinger et al.1992; Vogel RM 1993; Markiewicz et al. 2006; Mitosek et al. 2006; Cunnane., 1989; Ishak et al., 2010; Haddad et al., 2011; Haddad et al., 2012; Haddad and Rahman, 2012; Zaman et al., 2012; Haddad et al., 2013). However, due to the limited length of observed flood data as compared to the return period of interest, this becomes a challenging task and often FFA is associated with controversies (Bobee et al. 1993).

Selection of a probability distribution is of fundamental importance inFFA, as the wrong choice could lead to significant error and bias in design flood estimates (as high as 100% different to the optimum estimation), particularly at higher return periods, leading to either under- or over-estimation, which may have serious implications in practice (Rahman et al., 2013).

Several probability models are available to explain the distribution of AM flood data at a single site. However, the choice of a suitable probability model is still a major problem since there is no general agreement as to which distribution, or distributions is the best fit probability distribution for design flood estimaiton through FFA. Therefore, it is ideally necessary to evaluate many available distributions in order to find a suitable probability distribution that could provide more accurate design flood estimates (Tao et al., 2002). In ARR 1987, LP3 distribution coupled with method of product moments (MPM) was recommended for general use, similar to the USA (I. E. Aust., 1987; USWRC, 1967). However, the most recent version of ARR did not recommend any specific probability distribution for flood frequency analysis (Ball et al., 2016). Rahman et al. (2013) investigated the feasibility of 15 different probability distributions for Australia and found that a single distribution could not be specified as the best-fit distribution for all Australian states. They identified LP3, generalized extreme value (GEV), and generalized Pareto (GP) distributions as the top three best-fit distributions. Haddad and Rahman (2010) in their study found the two parameter Log Normal (LN) to be the best-fit distribution for Tasmania. Zhang et al. (2017) recommended GEV as the best statistical distribution for 34 stations in the Pearl River Delta during a period of about 60 years.

Various methods are used for estimating the parameters of a probability distribution. Methods of moments (MOM), maximum likelihood (MLE), L moments, LH moments and Bayesian methods are commonly used parameter estimation procedures. The MOM equates sample moments to parameter estimates. The product moments of a data series in MOM are equally influenced by low ovalues in data series same as by the higher observations. Also in MOM, the coefficient of variation and skewness are much affected by extremes in the data series. On the other hane L moments are less affected by extremes in the data series (Hosking 1990). The LH moments provide more weighting to the larger values in the flood series and hence are expected to provide better fits to the upper tail of the distribution (Wang 1997). MLE is an alternative to MOM and often statisticians give it preference over MOM (Bickel and Doksum 1977; Martins and Stedinger 2000). Bayesian inference is an alternative to MOM and MLE. In Bayesian inference both the likelihood function and the parameters to be estimated are described by probability distributions. Bayes produces a complete characterization of the parameter from the single dataset. The use of Bayes' theorem for combining prior and sample flood information was introduced by Bernier (1967). Many researchers have adopted bayesian approach in FFA(e.g., Halbert et al., 2016; Parkes et aa., 2016; Griffiths et al., 2017; Kuczera 1982, 1983a, b, 1999, Smith et al., 2015; Viglione et al., 2013; Liang et al., 2012).

In Australia, there has been a lack of studies comparing different probability distributions, in particular for the Brisbane River catchment, which has a known history of severe flooding. Hence, this study research is devoted to finding the best fit probability distribution for flood estimation at the Brisbane River catchment under stationary assumption.

## 2. Methodology

### *2.1 Study area and data*

The Brisbane River basin, Australia has been selected as the study area. It is located in the south-east corner of Queensland. The catchment of the Brisbane River system has an area of 13,570 $km^{2.}$ The Brisbane River catchment includes the sub-catchments of the Upper Brisbane, Stanley, Lockyer and Bremer Rivers. The Brisbane River is the largest river in the catchment. The Brisbane River is a large and complex river system. It has a long history of flooding with significant flood events in 1974 and 2011 that caused widespread damage. The Cooyar Creek, Emu Creek and Cressbrook Creek are the main tributaries of the upper Brisbane River. The Brisbane River catchment drains to Moreton Bay, a shallow bay sheltered from the Pacific Ocean by the islands of Moreton and North Stradbroke to the east. Figure 1 illustrates the Study area with seven main Brisbane River sub-catchments.

The Brisbane river system has many stream gauges. In this study, up-to-date continuous stream gauge recordings from the Department of Natural Resources and Mines (DNRM) is collected via the DNRM website. Data includes daily maximum

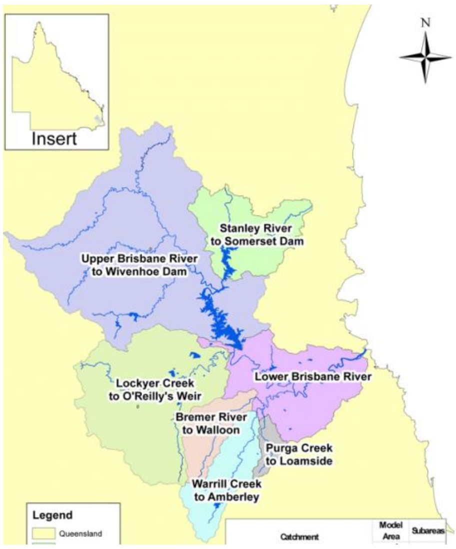


**Figure 2.** Location of the selected stream gauges in the Brisbane River Catchment

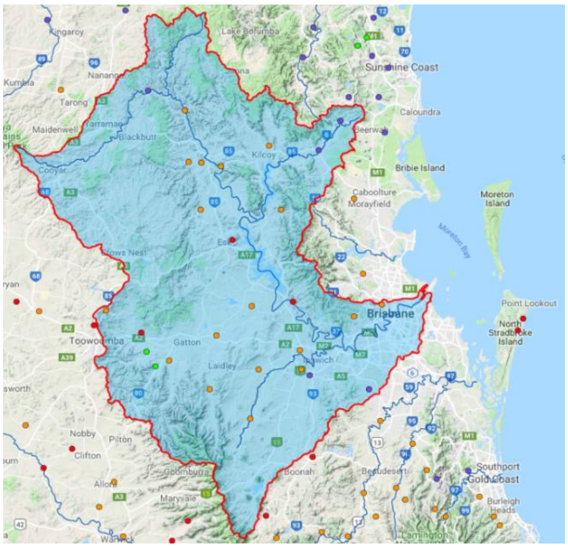


**Figure 1.** Location of the selected stream gauges in the Brisbane River Catchment

**Table 1**: Selected catchments with annual maximum flood record length

| Station ID | Description | Period of Record | Record Length (Year) |
|---|---|---|---|
| 143001C | Brisbane River at Savages Crossing | 1958-2017 | 60 |
| 143007A | Brisbane River at Linville | 1964-2017 | 54 |
| 143009A | Brisbane River at Gregors Creek | 1962-2017 | 56 |
| 143010B | Emu Creek at Boat Mountain | 1967-2017 | 51 |
| 143015B | Cooyar Creek at Taromeo Creek | 1969-2017 | 49 |
| 143028A | Ithaca Creek at Jason Street | 1972-2017 | 46 |
| 143032A | Moggill Creek at Upper Brookfield | 1976-2017 | 42 |
| 143033A | Oxley Creek at New Beith | 1976-2017 | 42 |
| 143107A | Bremer River at Walloon | 1961-2017 | 57 |
| 143108A | Warrill Creek at Amberley | 1961-2017 | 57 |
| 143110A | Bremer River at Adams Bridge | 1962-2015 | 54 |
| 143113A | Purga Creek at Loamside | 1973-2017 | 45 |
| 143203C | Lockyer Creek at Helidon Number 3 | 1927-2017 | 91 |
| 143207A | Lockyer Creek at O'Reillys Weir | 1948-2014 | 67 |
| 143209B | Laidley Creek at Mulgowie | 1967-2016 | 50 |
| 143210B | Lockyer Creek at Rifle Range Road | 1988-2017 | 30 |
| 143212A | Tenthill Creek at Tenthill | 1968-2017 | 50 |
| 143213C | Ma Ma Creek at Harms | 1995-2017 | 23 |
| 143219A | Murphys Creek at Spring Bluff | 1979-2017 | 39 |
| 143229A | Laidley Creek at Warrego Highway | 1990-2017 | 28 |
| 143232A | Sandy Creek at Forest Hill | 1995-2014 | 20 |
| 143233A | Flagstone Creek at Brown-Zirbels Road | 1995-2017 | 23 |
| 143303A | Stanley R at Peachester | 1927-2017 | 91 |
| 143306A | Reedy Creek at Upstream Byron Creek Junction | 1975-2011 | 37 |
| 143921A | Cressbrook Creek at Rosentretters Crossing | 1986-2015 | 30 |
| 143307A | Byron Creek at Causeway | 1975-2010 | 36 |

flow and AM flow time series. After preliminary investigation of the data series, particularly record length and the data quality, 26 stream gauging stations are selected for this study. Stations graded as 'poor quality' or with specific comments by the gauging authority regarding the quality of the data are assessed in greater detail; if they are deemed 'low quality' they are excluded. The network of these 26 gauging stations across the Brisbane River catchment are shown in Figure 1.

The average catchment area of the selected 26 stations is 1075 $km^2$. The majority of the gauge records are from the post-1960 period. All gauging records have flow records from the 2011 flood, one of the most recent severe flood events after 1974. The AM flood series record lengths of the selected 26 stations are in the range of 20 to 91 years, with a mean value of 47 years. Most of the stations show that the the highest peak flow event occured in 2011 during the recorded data period. Details of 26 selected stream gauging stations is shown in Table 1.

### 2.2 Method

Abstraction of raw data from the DNRM website and review of the available flow data is the first step of the methodology. The characteristics of the available flood data at the particular site is vital in the selection of the best probability distribution. To find a more appropriate probability distribution that is closer to the parent distribution, longer periods of observed flood data is reommended. However, at most of the stream gauging sites, measured data lengths are commonly shorter compared to return periods of interest. The minimum record length used in this study is 20 years. In this stady three-step methodology is adopted, i.e. (i) selection of candidate probability distributions; (ii) selection of appropriate parameter estimation methods; (iii) carrying out hypothesis testing to evaluate goodness-of-fit of the hypothesised probability distributions to the observed annual maximum flood (AMF) data, and applying selection criteria for choice of statistical distribution.

#### *2.2.1 Selection of candidate probability distributions*

A list of probability distributions applied in practice is summerised by Cunnane, 1989. AM flood data are often found to be skewed, which has led to the development and use of many skewed distributions in flood frequency analysis (Rahman et al., 2013). Based on the recommendations in relevant literature, five commenly used parametric distributions, i.e., LN, LP3, Gumbel, GP and GEV are selected for this study. The LP3 distribution as recommended in the Australian Rainfall and Runoff (I. E. Aust., 2001; Pilgrim, 2001) is included in this study. The GEV distribution which had been favoured by many recent studies is also included. The mathematical formulation of these probability distributions are available in many literatures (i.e. FLIKE, 2017). Two statistical software, EasyFit (Mathwave, 2017; Drokin, 2018) and FLIKE (Kuczera and Franks, 2016; Kuczera, 1999), are used in this study.

#### *2.2.2 Selection of candidate parameter estimation methods*

Estimation the parameters of the selected probability distributions using the selected flood data is the next step in selecting best-fit probability diatribution. This study uses number of parameters estimation procedures. EasyFit software used in this study uses the method of moments (MOM) for Gumbel and for LP3 distributions, maximum likelihood method (MLM) for LN and method of L-moments for GEV and for GP distributions. The FLIKE software includes Bayesian and L-moment fitting for all distributions including GEV, and for GEV, FLIKE includes additional LH-moment fitting. Both EasyFit and FLIKE softwares provide graphical fitting of the selected probability distributions, which provides a clear visual assessment of the fitted distributions to the given AM flood data.

#### *2.2.3 Selection of candidate goodness-of-fit tests*

To test whether a particular probability distribution provides an adequate fit to the observed flood data series, three different widely used goodness-of-fit tests are considered. ,The. Chi-squared (C-S) test, Kolmogorov–Smirnov (K-S) test, and Anderson–Darling (A-D) test, are adopted in this study. These tests calculate test-statistics are used to ascertain the suitability of a given distribution to fit the flood data. The goodness of fit tests are carried out using EasyFit, software (Mathwave, 2017; Drokin, 2018). EasyFit supports all popular goodness-of-fit tests, including the K-S, A-D, and C-S tests. Once the distributions are fitted, EasyFit displays the goodness-of-fit reports which included the test statistics and critical values calculated for various significance levels. These tests are briefly described below. Visual observation of the fitted distributions is done using FLIKE-produced plots and comparing with Easyfit results.

#### *2.2.3.1 Kolmogorov-Smirnov test*

Kolmogorov-Smirnov test is used to determine whether a sample has come from an assumed continuous probability distribution. For a detailed description of the test see Chakravarti et al. (1967). This test is based on the empirical cumulative distribution function (ECDF), which is given by:

$$F_n(x) = \frac{1}{n}.\left[Number\ of\ \ observations \le x\right] \tag{1}$$

The Kolmogorov-Smirnov test statistic (*D*) is given by the largest vertical difference between the theoretical and empirical cumulative distribution functions:

$$D = \max_{1 \le i \le n}\left( F(x_i) - \frac{i-1}{n}, \frac{i}{n} - F(x_i) \right) \tag{2}$$

$$D = max|(X_m) - F(X_m)| \tag{3}$$

Where $P(X_m) = \left( F(x_i) - \frac{i-1}{n}, \frac{i}{n} \right)$

*P(X_m)* is the cumulative probability distribution for each of the ordered observations $X_m$ using Weibull's formula, and *F(Xm)* is the theoretical cumulative probability for each of the ordered observations $X_m$ using the assumed distribution (Sharma et al., 2016). The large values of D indicate the presence of non-normality in the time series (Machiwal and Jha, 2012).

*2.2.3.2 Anderson-Darling test*

Anderson-Darling test compares the fit of an observed cumulative distribution function to an expected cumulative distribution function. This test gives more weight to the tails of the distribution than the Kolmogorov-Smirnov test. The A-D test has been used as an alternative to the K-S and Chi-Squared goodness-of-fit tests. The Anderson-Darling test statistic ($A^2$) is given by:

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1).\left[\ln F(X_i) + \ln(1 - F(X_{n-i+1}))\right] \tag{4}$$

Where, $X_i$, $X_{i+1}$, ,.,., Xn are data series, F = cumulative distribution function (CDF), and n = size of the sample The hypothesis that the distribution is normal is rejected if the value of A is greater than the critical value

*2.2.3.3 Chi-squared test*

Chi-squared test is used to find if a sample has come from a population with a given distribution. This is applied to the binned data, and hence the value of the test statistic depends on how the available data is binned. The Chi-squared test statistic is given by:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{5}$$

where $O_i$ is the observed frequency, *i* is the number of observations (1, 2, …, *k*) and $E_i$ is the expected frequency for bin *i* obtained by:

$$E_i = F(x_2) - F(x_1) \tag{6}$$

Where, *F* is the cumulative distribution function of the probability distribution being tested, and $x_1$, $x_2$ are the limits for bin *i*. Although there is no optimal choice for the number of bins (*k*), there are several formulas which can be used to calculate this number based on the sample size (*N*). The observed number of observations/bins (*k*) in interval '*i*' for sample size of *N* is computed by:

$$k = 1 + \log_2 N \tag{7}$$

The hypothesis that the data are from a population with the specified distribution is rejected,
if $\chi^2 > \chi^2_{(\alpha,\ \kappa\Box\chi)}$

Where, $\chi^2_{(\alpha,\ \kappa \square \chi)}$ is the critical test-statistic value with $k–c$ degrees of freedom and a significance level($\alpha$).

### 2.2. Selection of the best fit probability distribution

The three goodness-of-fit tests (as mentioned in Section 2.2.3) were applied to the AM flood data series at each of the selected 26 stations. The test statistics corresponding to each of these tests were computed and hypothesis testing was carried out at the 0.05 level of significance. The selected 5 probability distributions are ranked on a scale of 1 to 3 for all the three tests independently, with rank 1 indicating the best fit distribution, and rank 2 the second best one, and so on. The final selection was made based on total test scores derived by combining all the three goodness-of-fit tests. A maximum score of 3 was awarded to rank 1 probability distribution, and score 2 for rank 2 distributions, and score 1 for rank 3.

### 2.3. Graphical observation test

A visual inspection method or graphical test of the distribution can be used for assessing the goodness of fit tests. A graphical test is one of the most simple and powerful techniques for selecting the best-fit model The frequency distribution (histogram), stem-and-leaf plot, boxplot, P-P plot (probability-probability plot), and Q-Q plot (quantile-quantile plot) are used for visual checking. The frequency distribution plots the observed values against their frequency and provides both a visual judgment about whether the distribution is bell shaped, and insights about gaps in the data and outlying values (Ghasemi and Zahediasl, 2012). In this study, graphical testing is applied to compare the observed and estimated flood values. EasyFit software provides graphical (Q-Q plot) fitting of the selected probability distributions, which is used for visual comparison of the fitted distributions to the given data. FLIKE software is used for visual comparison of graphical quantile plot between plot of expected probability quantile and observed flow with annual exceedance probability (AEP) for each probability distribution which is used for visual comparison of the fitted distributions to the given data. The plots for different distributions were generated for visual assessment of the quality of fit. FLIKE plots of the observed AMF data and the fitted distributions were examined to make a visual assessment of the goodness-of-fit test results.

### 2.4. Flood quantile estimation

Probability distribution is used to estimate the quantile i.e. exceedance probability of a given value of *X* or alternatively to estimate the p-quantile of *X* (where p denotes the non-exceedance probability). FLIKE software is used in this study for flood quantile estimation. Flood quantiles for 2, 5, 10, 20, 50 and 100-year ARIs were estimated along with 90% confidence limits.

### 2.5. Sensitivity analysis

This study has carried out a sensitivity analysis of the flood quantile estimation and selection of the best-fit probability distribution by (i) removing the highest recorded flow of a station's AMF data series, (ii) removing both the first and second highest from the AMF data series and (iii) removing the first, second and third highest recorded flow from AMF data series. Parameter estimations, goodness of fit tests and selection of best-fit probability distribution, and quantile estimation are carried out for these three scenarios.

## 3. Results and discussion

### 3.1 Goodness of fit tests

Each of the 5 selected distributions is fitted to the AM flood data set at each of the 26 stations. The results of the A-D, K-S and C-S Goodness of fit (GoF) tests for Stations 143009A is summarised in Tables 2. To select the best-fit distribution, a comparative assessment of all five distributions at each site is carried out. Figure 3 shows a summary of GoF test results with rank 1 for selected stations. To identify the best fit probability distribution overall i.e. the distribution that fits the highest numbers of the selected stations, a relative scoring method based on the results given by the three goodness-of-fit tests is adopted. The scoring results of the distribution selection are summarised in Table 3.

The best fit probability distribution is identified based on the highest score that was determined based on the three goodness-of-fit tests. The combine score of the GoF test results in Table 3 shows that LP3 distribution is the most preferred probability distribution with score 179; followed by GP with score 114. It is also seen that Gumbel distribution is the least preferred probability distribution with score 16, as only the K-S GoF test selects this for only one station as rank 1.

**Table 2.** Summary of GoF test results for candidate probability distributions for Station 143009A

| Distribution | Kolmogorov Smirnov (K-S) | Kolmogorov Smirnov (K-S) | Anderson Darling (A-D) | Anderson Darling (A-D) | Chi-Squared (C-S) | Chi-Squared (C-S) | Avg. Rank |
|---|---|---|---|---|---|---|---|
| | Statistic | Rank | Statistic | Rank | Statistic | Rank | |
| **Log Pearson type III** | **0.07086** | **1** | **0.40106** | **1** | **0.86208** | **1** | **1.0** |
| Lognormal | 0.07529 | 2 | 0.42323 | 2 | 1.1246 | 2 | 2.0 |
| Generalised. Pareto | 0.15407 | 4 | 1.5218 | 3 | 3.5449 | 3 | 3.3 |
| Gen. Extreme Value | 0.14498 | 3 | 1.72 | 4 | 3.6029 | 4 | 3.7 |
| Gumbel | 0.30794 | 5 | 7.1698 | 5 | 14.587 | 5 | 5.0 |

***Bold value** indicates the best-fit probability distribution as per GoF tests*

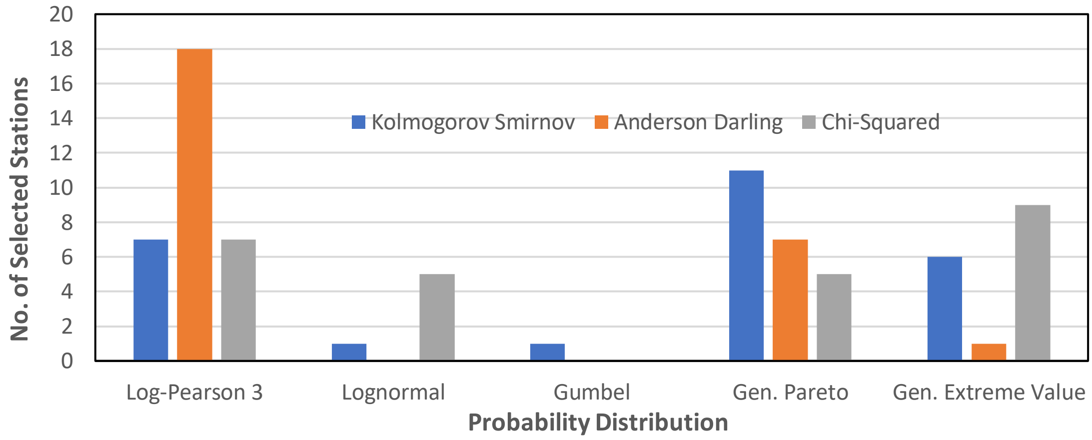


**Figure 3**. Summary of GoF tests for the 26 stations

**Table 3.** GoF results Combined Score (ranks 1, 2, and 3 for all stations with weights for rank 1, 2 and 3)

| Probability Distribution | K-S test | A-D test | C-S test | K-S test | A-D test | C-S test | K-S test | A-D test | C-S test | All Stations |
|---|---|---|---|---|---|---|---|---|---|---|
| Method | Number of stations with GoF test Rank **1** | | | Number of stations with GoF test Rank **2** | | | Number of stations with GoF test Rank **3** | | | Combine Scores |
| | Weight = 3 | | | Weight = 2 | | | Weight = 1 | | | |
| Log Pearson type III | 21 | 54 (69%) | 21 | 28 | 12 | 16 | 4 | 2 | 10 | 179 |
| Lognormal | 3 | 0 | 15 | 14 | 22 | 6 | 5 | 8 | 6 | 81 |
| Gumbel | 3 | 0 | 0 | 2 | 0 | 6 | 1 | 2 | 2 | 16 |
| Generalised. Pareto | 33 | 21 (27%) | 15 | 4 | 2 | 10 | 6 | 10 | 5 | 114 |
| Gen. Extreme Value | 18 | 3 | 27 | 4 | 16 | 14 | 10 | 4 | 3 | 104 |

The probability density functions (PDF) for the five probability distributions as shown in Figure 4 for stations 143028A show that LP3, GEV and LN distributions are most likely to fit best the observed AMF data. The histogram of AMF data reveals a positive skewed distribution and shows a unimodal distribution which is skewed to the right. The PDFs of the distributions are plotted to fit the empirical histograms of available records. The PDF shows that the Lognormal and LP3 distributions exhibit similar probability densities which are different from that of the GP and Gumbel distributions. The cumulative distribution function (CDF) (Figure 4b) shows the non-exceedance probability for a given magnitude. The probability-probability(P-P) plot (Figure 4c), which is a graph of the empirical CDF values plotted against the theoretical (fitted) CDF values, is used to determine how well a specific distribution fits the observed AMF data. It is recommended that if the maximum absolute difference is less than 0.05 (or 5%), the fit can be considered 'good.

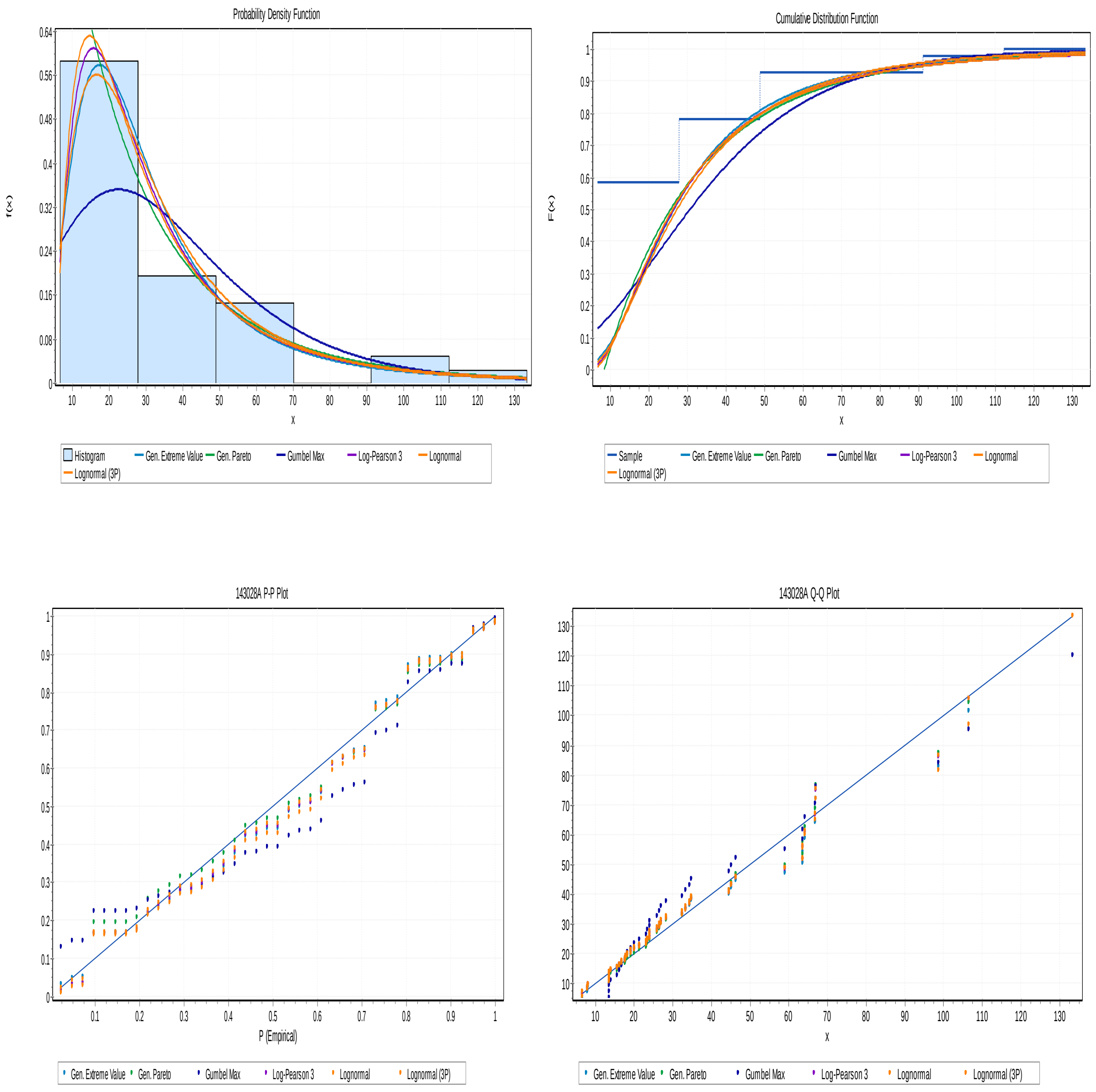


**Figure 4.** (a) Probability density functions (b) cumulative distribution functions, (c) probability-probability plots and (d) Q-Q plot for the five probability distributions for Station 143028A

### 3.2 Visual inspection and comparison with Goodness-of-fit test results

The best-fit probability distribution from the goodness of fit test is compared with graphical presentation from FLIKE which exhibits the AMF data and the fitted distributions. It is seen that the most appropriate probability distribution based on graphical observation does not fully agree with the goodness of fit test result for many stations. LP3 distribution is found to be the most preferred one according to the goodness of fit test results, followed by GP distribution as discussed in the earlier part of this chapter. However, according to graphical assessment, LP3 is the best-fit distribution for 9 stations only. The visual assessment of the AMF data and the fitted probability distributions for Station 143001C is shown in Figure 5. It can be seen from Figure 5 that for Station 143001C, LP3 is the best-fit probability distribution for quantile estimation. However, according to the goodness of fit test, GP is the best-fit distribution. Figure 6 show the plot AMF data estimated flood quantile for Station 143032A. It is seen in Figure 6 that for station 143010B, LP3 is the best-fit probability distribution as per A-D test and with visual inspection it is also LP3. These results highlight the importance of visual inspecting the data when selecting the best fit distribution for application.

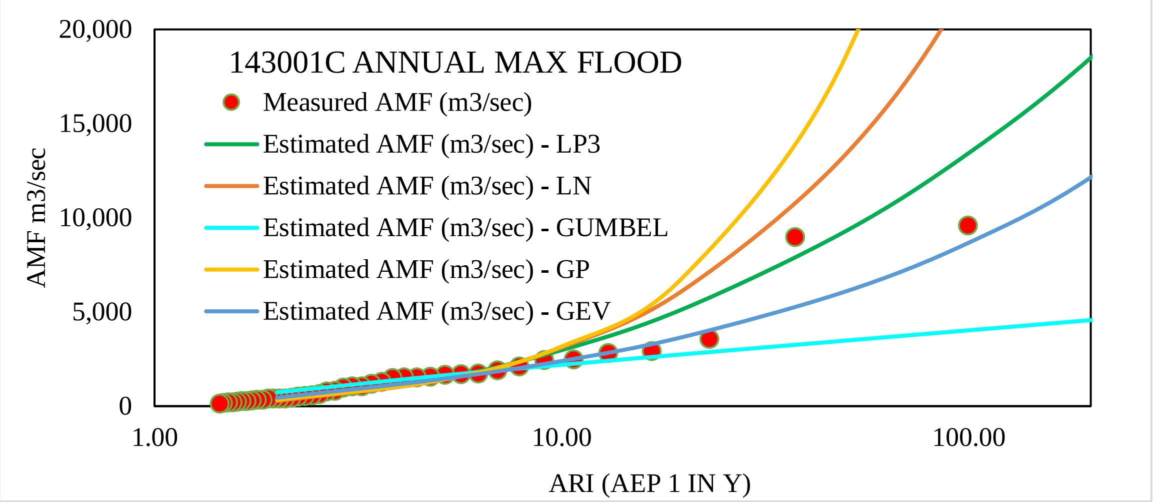


**Figure 5.** Comparison of flood quantile and AM flood data for five probability distributions for Station 143001C

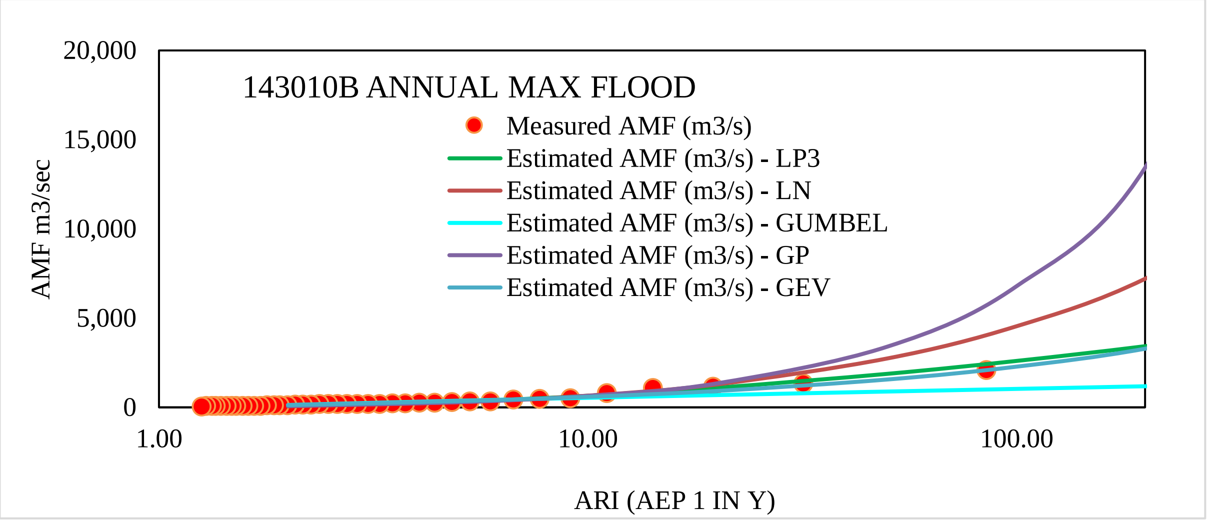


**Figure 6.** Comparison of flood quantile and AM flood data for five probability distributions for Station 143010B

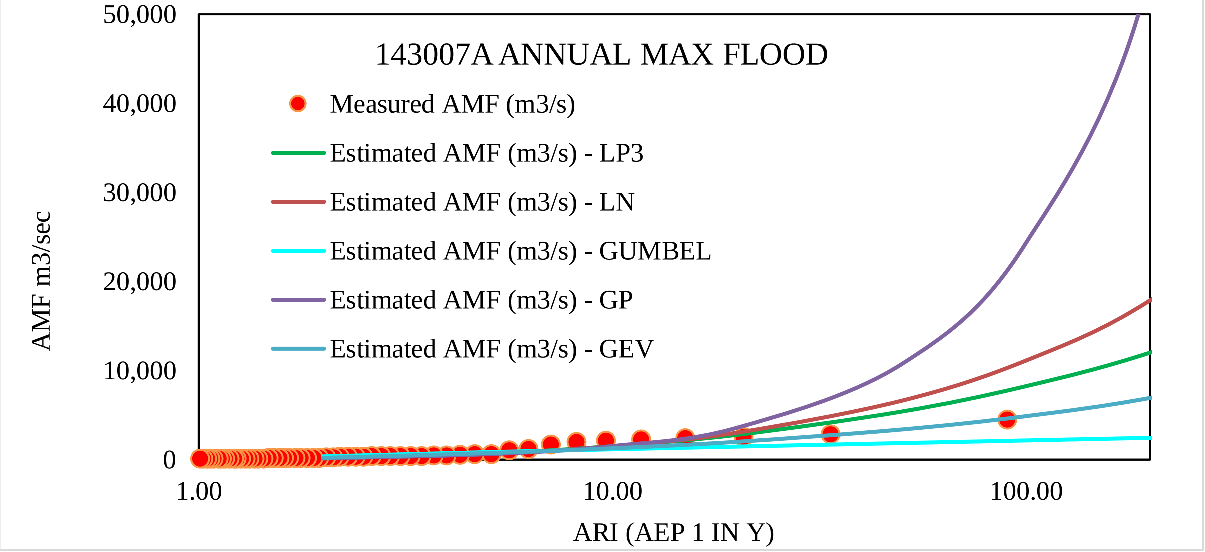


**Figure 7.** Comparison of flood quantiles using five different probability distributions for Station 143007A (period of record = 1964-2017)

### 3.3 Flood quantile estimation

Quantile estimation is done using FLIKE software corresponding to different return periods (for ARIs of 2, 5, 10, 20, 50 and 100 years). Figures 7 and 8 show quantile for all 5 distributions for stations 143007A and 143028A, respectively. Quantile plots (Figure 7 and 8) show that at low return periods, a good match is observed among observed AMF and quantile values. However, for higher return periods, especially 100 ARI or more, it becomes more difficult to choose the most preferred distribution. Table 4 shows the flood quantiles for 143010B with 5 different distributions. It is seen from the table that flood quantiles for 100-year ARI is quite different among the distributions and this is valid for all 26 stations. It is found that for almost all the stations, flood quantile estimates with Gumbel and Lognormal are notably different than that of LP3, GP and GEV distributions.

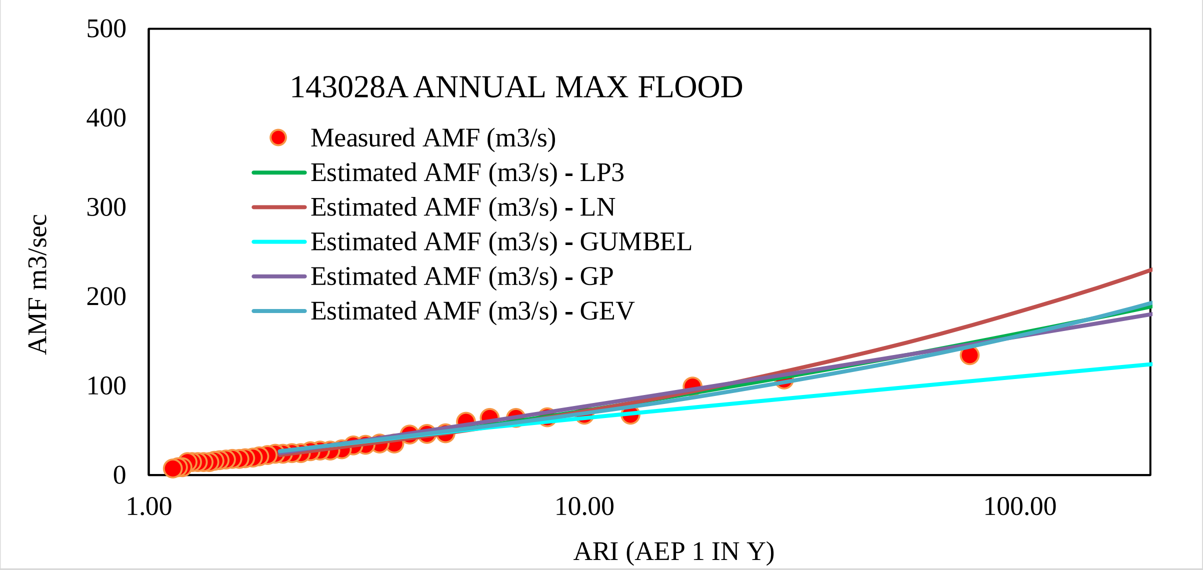


**Figure 8.** Comparison of flood quantiles using five different probability distributions for Station 143028A (period of record = 1972-2017)

**Table 4.** Flood Quantile estimation using 5 different probability distributions for station 143010B

| ARI (years) | Qunatile ($m^3$/s) - LP3 | Quantile ($m^3$/s) - LN | Quantile ($m^3$/s) - Gumbel | Quantile ($m^3$/s) - GP | Quantile ($m^3$/s) - GEV |
|---|---|---|---|---|---|
| 2 | 67 | 61 (91%) | 133 (197%) | 74 (111%) | 124 (185%) |
| 5 | 321 | 290 (90%) | 374 (117%) | 290 (90%) | 352 (110%) |
| 10 | 649 | 655 (101%) | 534 (82%) | 643 (99%) | 590 (91%) |
| 20 | 1099 | 1283 (117%) | 687 (63%) | 1341 (122%) | 918 (84%) |
| 50 | 1878 | 2738 (146%) | 886 (47%) | 3397 (181%) | 1560 (83%) |
| 100 | 2600 | 4537 (175%) | 1035 (40%) | 6771 (260%) | 2277 (88%) |
| 200 | 3427 | 7204 (210%) | 1183 (35%) | 13425 (392%) | 3286 (96%) |

Note: The % value is the difference of Quantile using 4 probability distributions and using LP3

Table 5 shows that the observed AMF values in 2011 ($Q_{2011}$) (a devastating flood occurred in 2011) were larger than estimated 100-year flood quantiles for 2 stations and 3 stations were similar to $Q_{2011}$, however, in 21 cases, the $Q_{2011}$ values were smaller than 100-year flood quantiles.

**Table 5:** Goodness of Fit test result summary of 26 stations excluding outliers in data

| Station | Observed $Q_{max}$ / $Q_{2011}$ ($m^3/s$) | Estimated Quantile with T=100; $Q_{100}$ ($m^3/s$) | % Difference (Quantile/Observed) |
|---|---|---|---|
| 143203C | 3643 | 1989 | 55 |
| 143219A | 362 | 348 | 96 |
| 143108A | 2108 | 2117 | 100 |
| 143303A | 710 | 721 | 102 |
| 143107A | 2057 | 2107 | 102 |
| 143113A | 411 | 434 | 106 |
| 143001C | 9533 | 10784 | 113 |
| 143028A | 133 | 159 | 119 |
| 143209B | 349 | 416 | 119 |
| 143207A | 2977 | 3582 | 120 |
| 143033A | 385 | 469 | 122 |
| 143010B | 2036 | 2600 | 128 |
| 143015B | 2335 | 3080 | 132 |
| 143306A | 175 | 231 | 132 |
| 143307A | 462 | 624 | 135 |
| 143210B | 1401 | 1958 | 140 |
| 143110A | 370 | 520 | 141 |
| 143232A | 45 | 63 | 141 |
| 143212A | 1359 | 2213 | 163 |
| 143032A | 297 | 533 | 179 |
| 143921A | 590 | 1058 | 179 |

### 3.4 Sensitivity analysis

Selection of best fit distribution may be changed if the highest flood record from the AMF data series is ignored. To investigate the sensitivity of the selection of the best-fit distribution and quantile estimation, FFA is carried out by removing the first highest flood record from the AMF data series, FFA is carried out by removing two highest flood records and FFA is carried out by removing three highest flood records. Figure 9 shows best-fit distribution with 3 different goodness-of-fit tests by removing the highest flood event from each of the 26 station's AMF data.

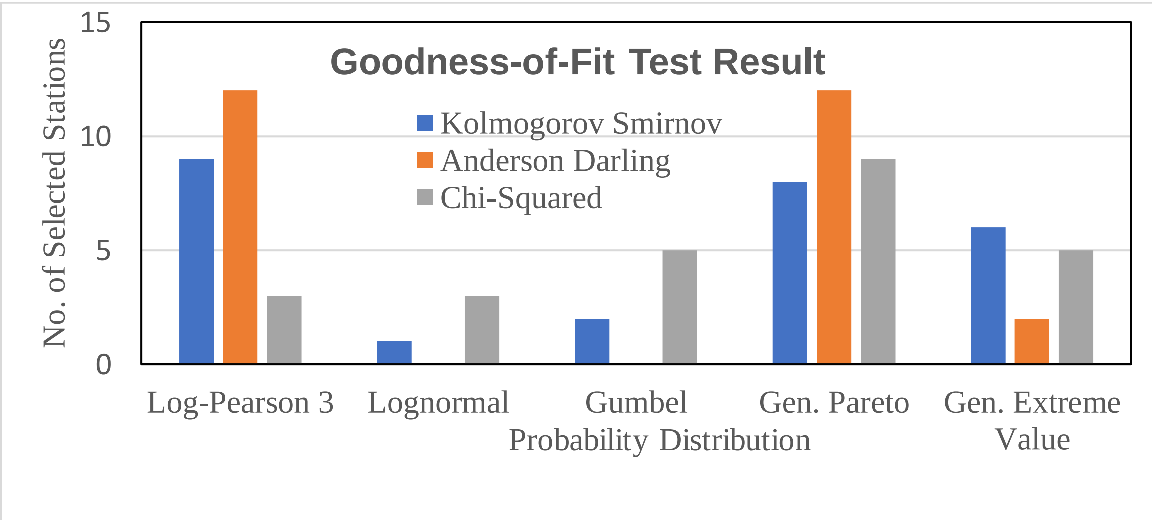


Figure 9. Summary of GoF tests for the 26 stations by removing the first highest, second highest and third highest AM flood data points

It is seen combined score that GP is the best fit distribution followed by LP3.As shown earlier that without removing highest records LP3 becomes the best fit distribution. Therefore presence of extreme records in the AMF time-series has influenced selection of distribution.

## 4. Conclusion

This paper examines the selection of the best fit probability distribution for annual maximum (AM) flood data in Brisbane River catchment. A total of 26 stations are used in this study. Missing data are very few in numbers (smaller than 2% cases) and are infilled through regression analysis. Presence of outliers in AM flood time series data have been tested using FLIKE software and all outliers in data are censored in flood frequency analysis. Five different probability distributions and three goodness-of-fit tests, Kolmogorov-Smirnov, Anderson-Darling and Chi-squared tests are adopted. It has been found that there is no single distribution that fits the AM data for all the 26 stations. Based on relative scoring method (Table 3), the LP3 distribution is found to be fitting the maximum number of stations with 69% (based on Anderson-Darling test) of the selected stations followed by GP distribution with 27% stations. Sensitivity analysis shows that the best fit probability distribution is sensitive to highest recorded AM flood data. Since the quantile estimates of higher ARIs are greatly influenced by skewness, a longer record length is desirable in order to reduce the uncertainty in higher quantile estimates. The AM flood data series for majority of the study stations show a positive skewness. In Australia, some previous studies (e.g. Srikanthan and McMahon, 1981; Rahman et al., 2013) find LP3 as the most favourable distribution for FFA since the skewness of logged AM flood data in Australia generally do not exceed the desirable limit of ± 1.4 (Rahman et al., 2016; Griffis and Stedinger, 2005, 2009).

Flood quantile is estimated for every station using all the five probability distributions. It appears (Table 5) 100- year quantile estimation with LP3 distribution for majority of the stations are within 95%-140% of the maximum flood value in the respective AMF data series. It has been found that the best-fit probability distribution can change if FFA is made by removing maximum recorded flow form the AMF data series.

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