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Modal analysis of cracked shafts

Erfan Taheri, Helen Wu and Ming Zhao
School of Computing, Engineering and Mathematics, Western Sydney University, Australia.
*Corresponding author's email: 17783747@student.westernsydney.edu.au

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Abstract: *The mechanical shaft is a key component in most mechanical systems. It is in charge of transferring power from one part of the machine to another part. As the shaft spins over long operational hours it may become prone to forming a fatigue crack. Often cracks show no symptoms until the shaft reaches failure. It is with utmost importance to develop a method to detect a cracked shaft before it reaches failure. Modal analysis is the first step to reach the goal and understand the vibration behaviour of a machine. In this work, Finite Element analysis is performed to acquire the mode shapes from cracked shaft models developed through Abaqus. The models include various shafts, such as an intact shaft, a shaft with varying crack depth ratios [0.5, 1, and 1.5], crack locations and the number of cracks [1 and 2 cracks]. Each shaft model developed will have 5 modes tested and the natural frequency will be derived for each mode. Comparison of the frequencies shows that shaft stiffnesses are significantly reduced for a shaft with a larger crack, and also a shaft with more cracks. The results gathered are new and provide a contribution to rotor-dynamics for eventual development of a method for early crack detection.*

***Keywords**: Crack; Shaft; Rotor; Frequency.*

1. Introduction

Almost all mechanical machines in the fields of transportation and manufacturing require a shaft. As the shaft is rotating at high speeds, it faces internal pressures and quick changes in bending moment stresses. These pressures and stresses may lead to a shaft developing a fatigue crack. When a shaft initially develops a crack, it may exhibit no symptoms until it is too late, and reaches failure (when the shaft splits into two or more pieces). Shaft failure could lead to financial loss and be potentially lethal to humans by causing injuries or fatalities (e.g. shaft failure in an operating aircraft). There are currently no known methods for early crack detection while a machine is in operation (dynamic movement). Therefore, this field is of high importance to rotor dynamic studies as finding a solution will prevent loss of business downtime, injuries and fatalities.

The modal and vibrational signature of a shaft without a crack could be measured and graphed, hence the characteristics of a "normal" shaft could be gathered. Then the vibration on a shaft with a crack could be measured to compare against the non-cracked shaft. Multiple parameters are incorporated, such as the crack's depth (ratio of the cracks length relative to the shafts diameter), crack's location (the location of the crack with respect to the shaft) and adding multiple cracks. The aim will be to first develop a simulation of a shaft with a crack and then to test all the parameters. This will yield radically new results that will significantly contribute to the literature of early crack detection. A shaft with fixed ends and two discs will be used. The exact dimensions of the shaft with its properties is presented in the research methodology section of this paper.

Mechanical systems are prone to resonance which is frequency that can eventually cause damage to the structure. By being able to measure the frequencies, which cause a structure to resonate, it would be possible to understand the effects of different frequencies on the structure. Modal testing involves finding the modes of vibration of whatever is being measured.

Sekhar (2004) conducted research based on a modal analysis to identify cracks in a rotor. The approach that was carried out in this study was the modal-based method which was applied to identify the crack in the rotor at a steady speed. A model-based technique by (Jain et al., 2003) was used for the crack identification, in which the full system state is reconstructed from a number of measured quantities by an observer during the iterative process. For the different crack locations and depths at different rotor speeds, it was found that the model based identification technique with modal expansion was successful. While the cause of the cracks was ascertained using the Fast Fourier method, the effectiveness of the identification process depends to a good extent on the number of measured locations (DOF). Modal analysis of a rotating shaft with a crack can give detailed knowledge on the nature of the structure. Through finding the modal shapes of a non-cracked shaft and then comparing it to a shaft with cracks, (with varying parameters) it can find distinguished characteristics and aid in early diagnostics of crack detection.

A material's mass, stiffness factor and damping value affect the material's modes. Every mode has a correlating frequency and a shape. Therefore, if the properties of a material are altered (i.e. a crack or varying crack parameters), then the modes will also change. A modal shape represents the oscillation of a component based on its frequency.

Multiple mode shapes can be present for the same component and thus each specific mode shape is related to its individual characteristic of its natural frequency.

The research will be done analytically by producing a shaft and testing several of the parameters mentioned above. The data collected will be tabled and graphed by the mode shapes and frequency. The results will be of significant value as it is narrowing the gaps to eventually developing a method of early crack detection during dynamic operation.

2. Methodology

2.1 Modelling cracked shaft

In order to test the modal and vibrational effects on a shaft with a crack, it is important to first model the system in a software and then to run simulations from it. A Finite Element Analysis software (FEA), Abaqus by Dassalt systems, was used to perform this task. Once the models are designed then a simulation is performed to obtain the modal signatures and corresponding frequencies for the cracked shaft models. Table 1 below shows the specifications for the shaft which was developed on Abaqus.

Table 1. Shaft specifications for development on Abaqus

Component	Measurements
Length of shaft	724 mm
Radius of shaft	6.35 mm
Inner radius of disk	6.35 mm
Outer radius of disk	54.5 mm
Crack locations	Variable
Distance to disk 1	181 mm on the positive x-axis
Distance to disk 2	543 mm on the positive x-axis
Weight of shaft (Gravity)	$9.81x10^{3}$ mm/s²
Mass of disk 1	0.5 kg
Mass of disk 2 (Additional mass added to create unbalancing force effect on the shaft)	0.5 kg + Variable (unbalancing force mass)
Stress	$210x10^{3}$ (N/mm²)
Density of shaft	$7.8x10^{-9}$ (kg ³)/mm³)

To create a shaft, which can simulate a crack, it has to have conditions that replicate real life conditions (with friction being taken into account, effect of gravity etc.). Abaqus was used to develop a shaft and the methodology lists the main steps that were undertaken in order to create such a shaft. Essentially there were three separate parts that were made, the shaft, the discs and the cracked part. After the parts were made, they were assembled together and the ends became fixed supports. The material, density of the shaft and poisson's ratio were all implemented into the material design (as per the parameters above in Table 1). Mesh was added to the component so the effects could be visualised and finer mesh was applied to the area of the crack. Figure 1 below shows the mesh used for the shaft prior to extrusion. The mesh size chosen was employed using elements which use eight node linear bricks, and thus the meshes density increases near the crack location for accurate results (Mobarak et al., 2017). The mesh value around the crack is $\Delta x_{crack} = 0.04$ for the measurements of the shaft's radius (listed in Table 1) and length of 5mm extruding to both directions from the crack. For every other component in the model, the mesh size $\Delta x_{other} = 0.2$, extending along the transversal and longitudinal direction of the shaft with measurements also in Table 1. Once the shaft is assembled it is possible to do modal analysis. 5 modes were selected to compare the results. Modal analysis to obtain the frequencies was done to a shaft without a crack, a shaft with a crack (with three different crack depths) and a shaft with varying crack locations (three locations; the centre of the shaft at 362 mm from either end and a crack near the left support before disc 1 and a crack to the right of disc 2 near the fixed end, both 90 mm inwards respective from the fixed ends). The final set up is shown below in Figure 2.

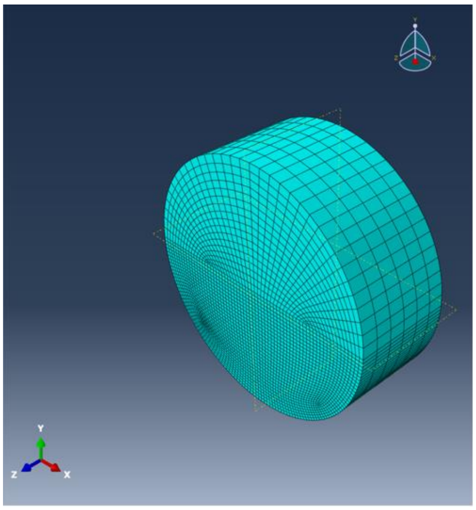


Figure 1. Abaqus modelled shaft showing mesh

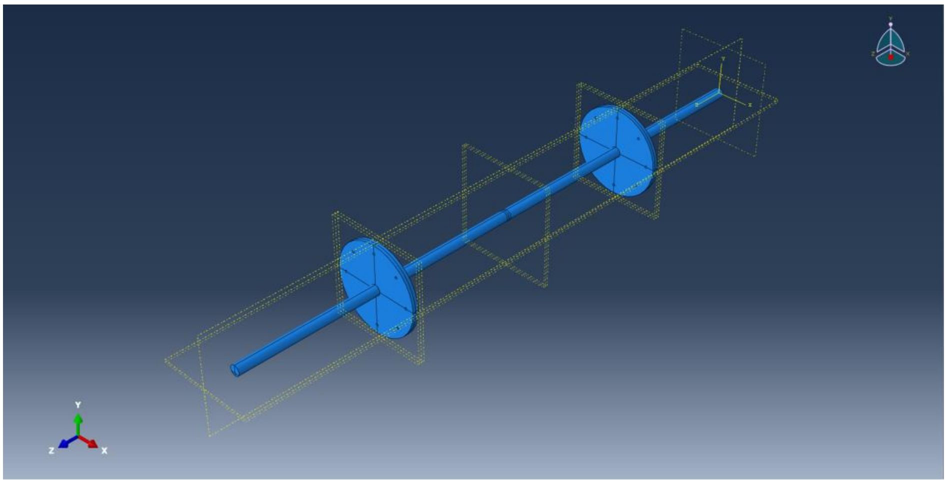


Figure 2. Abaqus model of shaft set up

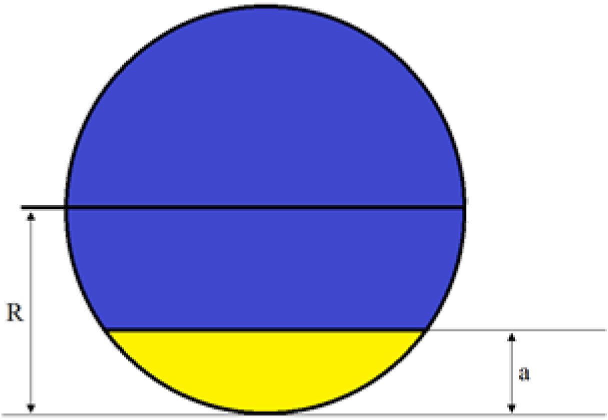


Figure 3. Diagram representing crack depth ratio

2.2 Shaft parameters

The crack depth ratio is the ratio between the crack's depth with respect to the radius of the shaft $(\mu = \frac{a}{R})$, as seen in Figure 3. There will be experimentation of a shaft with no crack followed with three crack ratios as shown below. The three crack depth ratios used for the preliminary results are μ = 0.5, 1 and 1.5. Figure 4 represents the crack's location with respect to the shaft. There are three different crack locations used for gathering the preliminary results. A crack located in the centre of the shaft, Lc2.The crack near the left fixed support at 90mm (from positive x axis), Lc1, and a crack on the right support at 634mm (from the positive x axis), Lc3. The cracks near the fixed ends are also on either side of the discs, with a crack depth ratio of 1. The effect that multiple cracks have on the modal frequency will be investigated. The control is a shaft with no crack. Then a shaft with one crack can be compared to a shaft two cracks (with crack depth ratio equal to 1), in two locations at the same time (Lc1 & Lc2).

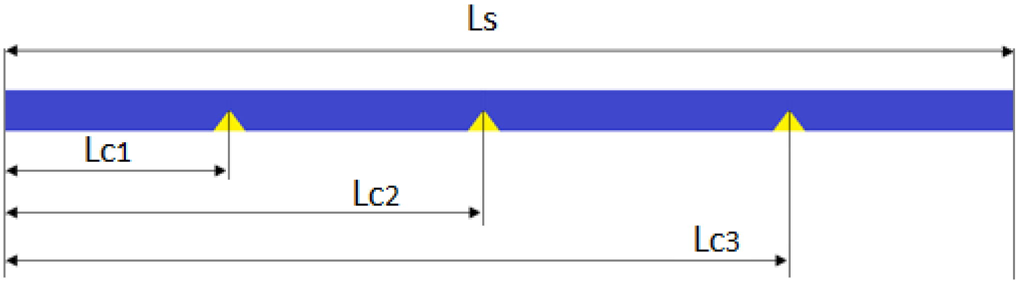


Figure 4. Crack locations

3. Results and discussion

Modal analysis using five nodes was conducted on a shaft with no crack (as the control), three different crack depth ratios ($\mu = 0.5, 1$ *and* 1.5), three different crack locations (Lc1, Lc2 and Lc3) and with multiple cracks (Lc1 and Lc2) . Table 2 shows the first mode signatures for a shaft with no crack, and the three crack depth ratios. The full results gathered are listed in Tables 3,4 & 5.

When comparing the effects that the crack depth ratio has on the shaft, it' noted that as the crack depth ratio increases then the frequency decreases. Having the same crack depth ratio but in different locations has minimal effect on the frequencies. This can be attributed to the symmetrical spreading of the crack and same crack depth ratio. When there were multiple cracks, then the frequency was decreased more than a single crack or no cracks.

Tables 3,4 & 5 showed values of frequencies with respect to the modes. The frequency formula $(\omega = 2\pi f = \sqrt{\frac{K}{m}})$ can be used to understand what is happening to the frequency when there is a crack presented.

Table 3. Results of modal comparison of different crack depth ratios (μ = 0.5, 1, 1.5)

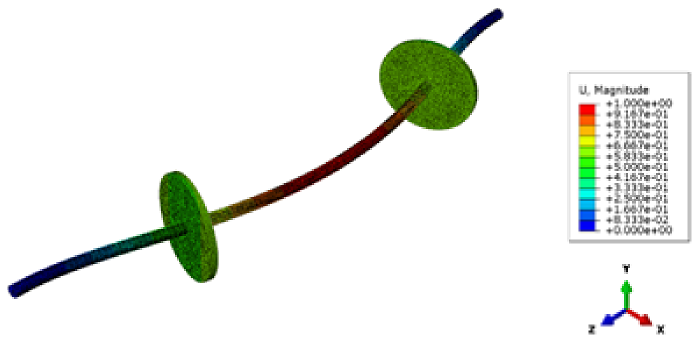
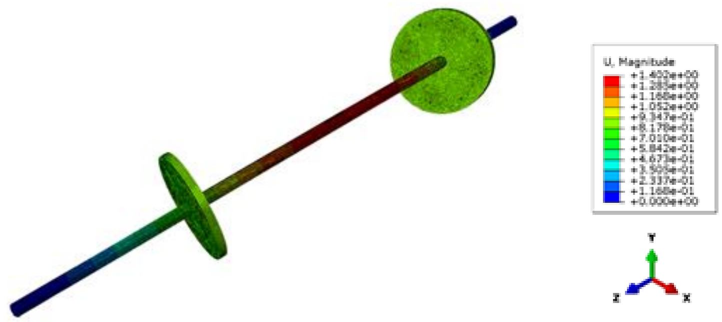
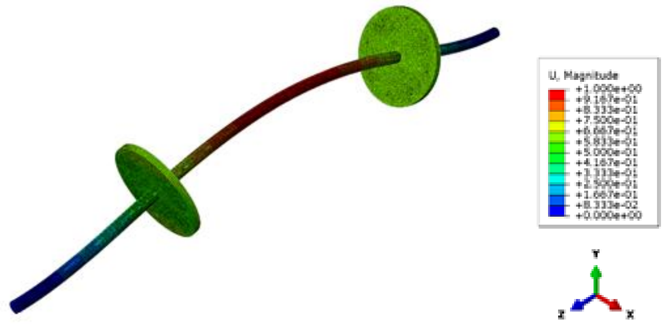
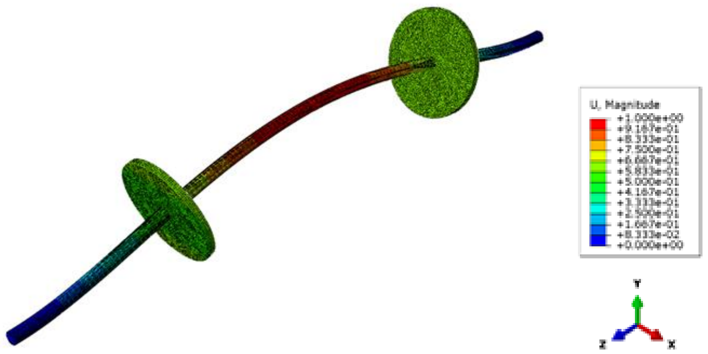
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
No crack	24.139 Hz	24.181 Hz	49.077 Hz	49.154Hz	62.278 Hz
Crack at μ = 0.5	24.096 Hz	24.129 Hz	49.079 Hz	49.154 Hz	62.202 Hz
Crack at μ = 1	23.595 Hz	23.903 Hz	48.189 Hz	48.530 Hz	61.939 Hz
Crack at μ = 1.5	23.321 Hz	23.790 Hz	47.727 Hz	48.243 Hz	61.931 Hz

Table 4. Results of Modal comparison of different crack locations

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Crack center of shaft- depth of 362mm on shaft	24.096 Hz	24.129 Hz	49.079Hz	49.154 Hz	62.202 Hz
Crack right of disc 2- depth of 634 mm on shaft	24.096 Hz	24.129 Hz	49.077 Hz	49.154 Hz	62.277 Hz
Crack left of disc 1 - depth of 90 mm on shaft	24.097 Hz	24.129 Hz	49.077 Hz	49.155 Hz	62.287 Hz

The frequency (f) is related to (K- stiffness factor) and (m - mass of the object). The mass remains constant whether the shaft has cracks or no cracks so the variable which effects the frequency is K. If a crack is present then there is a loss of stiffness as opposed to a shaft not having a crack. By simply analysing the formula it is possible to postulate that a decrease in K will thus cause a decrease in the frequency. The preliminary results seem to concur with this, a shaft with a bigger crack or multiple cracks, reduces the stiffness to a greater extent, which consequently causes a decrease in the frequency. Conversely, if the shaft has no cracks, the stiffness is higher and the frequency remains higher than shafts with a crack.

Table 2. Mode signatures for shafts

Shaft type	Frequency (Hz)	
Shaft with no crack (Mode 1)	24.139 Hz	
Shaft with crack at μ = 0.5 (Mode 1)	24.096 Hz	
Shaft with crack at μ = 1 (Mode 1)	23.595 Hz	
Shaft with crack at μ = 1.5 (Mode 1)	23.321 Hz	

4. Conclusion

The preliminary findings show that an analytical approach for modal analysis was accomplished by using Abaqus. A cracked shaft was successfully designed followed by the modal analysis conducted by analysing 5 modes and investigating the natural frequencies across the modes with varying crack parameters. Modal analysis was successful in identifying natural frequencies at different crack depths, locations and by adding multiple cracks. The data obtained identifies the relationship between the shaft's natural frequency to the crack's depth ratio, crack location and number of cracks, and can serve to assist further research in this field to eventually find a method for early crack detection during rotary operation.

Table 5. Results of Modal comparison of the number of cracks

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
No crack	24.139 Hz	24.181 Hz	49.077Hz	49.154Hz	62.278 Hz
One Crack with $\mu = 1$	23.595Hz	23.903 Hz	48.189Hz	48.530 Hz	61.939 Hz
Two Cracks with $\mu = 1$. Cracks at 90mm and 362mm	23.323 Hz	23.791 Hz	47.733 Hz	48.245 Hz	61.942

5. Acknowledgements

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